Examen aritmética 1º Bachillerato 16/10/15

① a) $\sqrt[3]{a^2bc}\cdot\sqrt[5]{ab^2c}=\sqrt[15]{a^{10}b^5c^5\cdot a^3b^6c^3}=\sqrt[15]{a^{13}b^{11}c^8}$

b) $\sqrt[3]{4\sqrt[3]{6\sqrt{8}}}=\sqrt[3]{\sqrt[3]{4^3 6\sqrt{8}}}=\sqrt[9]{\sqrt{4^6\cdot 6^2\cdot 8}}=\sqrt[18]{2^{12}\cdot 2^2\cdot 3^2\cdot 2^3}=\sqrt[18]{2^{17}\cdot 3^2}$

c) $3\sqrt{ab}+\sqrt{4a^3b}-2\sqrt{0'25ab}-a\sqrt{ab}-4\sqrt{\frac{1}{4}ab}=$

$=3\sqrt{ab}+2a\sqrt{ab}-\frac{2\cdot 5}{10}\sqrt{ab}-a\sqrt{ab}-\frac{4}{2}\sqrt{ab}=\cancel{3\sqrt{ab}}+2a\sqrt{ab}\cancel{-2\sqrt{ab}}-a\sqrt{ab}=$

$=\cancel{(2+a)\sqrt{ab}}-2\sqrt{ab}=a\sqrt{ab}.$

d) $\sqrt{\frac{2}{5}}-4\sqrt{\frac{18}{125}}+\frac{1}{3}\sqrt{\frac{8}{45}}=\sqrt{\frac{2}{5}}-\frac{4\cdot 3}{5}\sqrt{\frac{2}{5}}+\frac{1}{3}\cdot\frac{2}{3}\sqrt{\frac{2}{5}}=-\frac{12}{5}\sqrt{\frac{2}{5}}+\frac{2}{9}\sqrt{\frac{2}{5}}+\sqrt{\frac{2}{5}}=$

$\left(-\frac{12}{5}+\frac{2}{9}+1\right)\sqrt{\frac{2}{5}}=\frac{-108+10+45}{45}\sqrt{\frac{2}{5}}=\frac{-53}{45}\sqrt{\frac{2}{5}}$

② a) $\log_x 16=4\rightarrow x^4=16\Rightarrow x^4=2^4\Rightarrow\boxed{x=2}$

b) $\log_3 x=4\rightarrow 3^4=x\Rightarrow\boxed{x=81}$

c) $\log x+\log 30=1\Rightarrow\log x\cdot 30=1\Rightarrow 10^1=30x\Rightarrow\boxed{x=\frac{1}{3}}$

d) $\log(2x)=\log 32-\log x\Rightarrow\log(2x)=\log\frac{32}{x}\Rightarrow 2x=\frac{32}{x}\Rightarrow 2x^2=32$

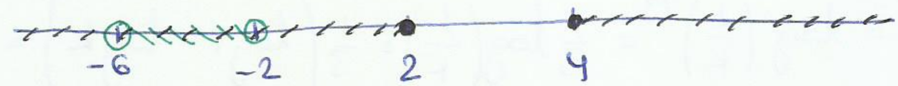
$x^2=16\Rightarrow\boxed{x=4}$

③ a) $|x-3|\geq 1\Rightarrow x-3\geq 1\Rightarrow x\geq 4$ $\quad(-\infty,2]\cup[4,+\infty)$

$x-3\leq -1\Rightarrow x\leq 2$

b) $|x+4|<2\Rightarrow -2<x+4<2\Rightarrow -6<x<-2$ $\quad(-6,-2)$

c)



$(-6,-2)$

④ a) $\frac{2\sqrt{a+b}}{\sqrt{a+b}-\sqrt{a-b}}\cdot\frac{\sqrt{a+b}+\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}=\frac{2\sqrt{a+b}\left(\sqrt{a+b}+\sqrt{a-b}\right)}{a+b-(a-b)}=$

$=\frac{2\sqrt{a+b}^2+2\sqrt{a+b}\sqrt{a-b}}{2b}=\frac{\cancel{2}(a+b)+\cancel{2}\sqrt{a^2-b^2}}{\cancel{2}b}=\frac{(a+b)+\sqrt{a^2-b^2}}{b}$

b) $\frac{1+a}{\sqrt[5]{a^3}}\cdot\frac{\sqrt[5]{a^2}}{\sqrt[5]{a^2}}=\frac{(1+a)\sqrt[5]{a^2}}{a}$

c) $\frac{-\sqrt{2}}{\sqrt[3]{2}(\sqrt{125}+2)} \cdot \frac{\sqrt[3]{2^2}(\sqrt{125}-2)}{\sqrt[3]{2^2}(\sqrt{125}-2)} = \frac{-\sqrt{2}\cdot\sqrt[3]{2^2}(\sqrt{125}-2)}{2(125-4)} = \frac{-\sqrt[6]{2^3 \cdot 2^4}(\sqrt{125}-2)}{2\cdot 121} =$

$= -\frac{2\sqrt[6]{2}(\sqrt{125}-2)}{2\cdot 121} = \frac{-\sqrt[6]{2}(\sqrt{125}-2)}{121}$ → Se puede hacer la operación del numerador.

⑤ a) $\log \frac{a^3\sqrt[5]{a^2b^4}}{b^2\sqrt[3]{a^5b}} = \log a^3\sqrt[5]{a^2b^4} - \log b^2\sqrt[3]{a^5b} =$

$= \log a^3 + \log\sqrt[5]{a^2b^4} - \left(\log b^2 + \log\sqrt[3]{a^5b}\right) =$

$= 3\log a + \log(a^2b^4)^{1/5} - \left[2\log b + \log(a^5b)^{1/3}\right] =$

$= 3\log a + \frac{1}{5}\log a^2b^4 - \left[2\log b + \frac{1}{3}\log(a^5b)\right] =$

$= 3\log a + \frac{1}{5}(\log a^2 + \log b^4) - \left[2\log b + \frac{1}{3}(\log a^5 + \log b)\right] =$

$= 3\log a + \frac{1}{5}(2\log a + 4\log b) - \left[2\log b + \frac{1}{3}(5\log a + \log b)\right] =$

$= 3\log a + \frac{2}{5}\log a + \frac{4}{5}\log b - 2\log b - \frac{5}{3}\log a - \frac{1}{3}\log b =$

$= \left(3+\frac{2}{5}-\frac{5}{3}\right)\log a + \left(\frac{4}{5}-2-\frac{1}{3}\right)\log b = \left(3+\frac{2}{5}-\frac{5}{3}\right)\cdot 2 + \left(\frac{4}{5}-2-\frac{1}{3}\right)\cdot 3 =$

$= \frac{45+6-25}{15}\cdot 2 + \frac{12-30-5}{15}\cdot 3 = \frac{52}{15} - \frac{69}{15} = \frac{-17}{15}$

b1) $\log\sqrt[3]{\frac{1}{k}} = \log\left(\frac{1}{k}\right)^{1/3} = \frac{1}{3}\log\left(\frac{1}{k}\right) = \frac{1}{3}(\log 1 - \log k) = -\frac{1}{3}\log k$

$= -\frac{1}{3}\cdot 14'4$

b2) $\log 0'1k^2 = \log 0'1 + \log k^2 = \log\frac{1}{10} + 2\log k = -1 + 2\cdot 14'4$.