(1) $\int \cos x \,\text{sen}^3 x \, dx$

(2) $\int \frac{x\,dx}{\sqrt{x^2+5}}$

(3) $\int \frac{2x+4}{(x-1)^2(x+3)} dx$

(4) $\int x^2 \ln x \, dx$

(5) $\int \frac{1}{x} \ln^3 x \, dx$

(6) $\int x \,\text{sen}(x^2) dx$

(7) $\int \frac{1-\text{sen}x}{x+\cos x} dx$

(8) $\int x \cdot 2^{-x} dx$

(9) $\int \frac{x^4+2x-6}{x^3+x^2-2x} dx$

(10) $\int \text{Ln}\, x \, dx$

(1) $\int \cos x \,\text{sen}^3 x\, dx = \begin{pmatrix} t = \text{sen}x \\ dt = \cos x\, dx \\ dx = \frac{dt}{\cos x} \end{pmatrix} = \int \cancel{\cos x} \cdot t^3 \cdot \frac{dt}{\cancel{\cos x}} = \frac{t^4}{4} = \frac{\text{sen}^4 x}{4} + K$

(2) $\int \frac{x\,dx}{\sqrt{x^2+5}} = \begin{pmatrix} t = x^2+5 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{pmatrix} = \int \frac{\cancel{x}}{\sqrt{t}} \cdot \frac{dt}{2\cancel{x}} = \frac{1}{2}\int t^{-1/2} dt = \frac{1}{2} \frac{t^{1/2}}{1/2} = \sqrt{x^2+5} + K$

(3) $\int \frac{2x+4}{(x-1)^2(x+3)} dx = \int \frac{1/8}{x-1} dx + \int \frac{3/2}{(x-1)^2} dx + \int \frac{-1/8}{x+3} dx = \frac{1}{8} \ln|x-1| - \frac{3}{2(x-1)} - \frac{1}{8}\ln|x+3| + K$

$\frac{2x+4}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{A(x-1)(x+3) + B(x+3) + C(x-1)^2}{(x-1)^2(x+3)}$

$x=1 \rightarrow 6 = 4B; \quad B = \frac{3}{2}$

$x=-3 \rightarrow -2 = 16C; \quad C = -1/8$

$x=0 \rightarrow 4 = -3A + 3B + C; \quad 4 = -3A + \frac{9}{2} - \frac{1}{8}; \quad 4 - \frac{9}{2} + \frac{1}{8} = -3A;$

$\frac{32-36+1}{8} = -3A; \quad -\frac{3}{8} = -3A; \quad A = 1/8$

(4) $\int x^2 \ln x\, dx = \begin{pmatrix} u = \ln x & du = \frac{dx}{x} \\ dv = x^2 dx & v = \frac{x^3}{3} \end{pmatrix} = \frac{x^3}{3}\ln x - \int \frac{x^3}{3} \cdot \frac{dx}{x} = \frac{x^3}{3}\ln x - \int \frac{x^2 dx}{3} = \frac{x^3}{3}\ln x - \frac{x^3}{9} + K$

(5) $\int \frac{1}{x} \ln^3 x\, dx = \begin{pmatrix} t = \ln x \\ dt = \frac{1}{x} dx \\ dx = x\,dt \end{pmatrix} = \int \frac{1}{\cancel{x}} \cdot t^3 \cdot \cancel{x}\, dt = \frac{t^4}{4} = \frac{\ln^4 x}{4} + K$

(6) $\int x\,\text{sen}(x^2) dx = \begin{pmatrix} t = x^2 \\ dt = 2x\,dx \\ dx = \frac{dt}{2x} \end{pmatrix} = \int \cancel{x}\,\text{sen}t \frac{dt}{2\cancel{x}} = \frac{1}{2}\int \text{sen}t\, dt = -\frac{\cos t}{2} = -\frac{\cos(x^2)}{2} + K$

(7) $\int \frac{1-\text{sen}x}{x+\cos x} dx = \text{Ln}|x+\cos x| + K$

(8) $\int x \cdot 2^{-x} dx = \begin{pmatrix} u = x & du = dx \\ dv = 2^{-x} dx & v = \frac{-2^{-x}}{\ln 2} \end{pmatrix} = -\frac{x \cdot 2^{-x}}{\ln 2} + \int \frac{2^{-x}}{\ln 2} dx = \frac{-x \cdot 2^{-x}}{\ln 2} - \frac{2^{-x}}{(\ln 2)^2} + K$

9) $\displaystyle\int \frac{x^4+2x-6}{x^3+x^2-2x}\,dx = \int\left(x-1+\frac{3x^2-6}{x^3+x^2-2x}\right)dx = (*)$

$$\begin{array}{l|l} x^4+2x-6 & x^3+x^2-2x \\ \hline -x^4-x^3+2x^2 & x-1 \\ \hline -x^3+2x^2+2x-6 & \\ +x^3+x^2-2x & \\ \hline 3x^2-6 & \end{array}$$

$$\frac{3x^2-6}{x^3+x^2-2x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2} = \cancel{A(x-1)(x+2)+Bx(x+2)}$$

$x(x^2+x-2)=0$

$x^2+x-2=0$

$x = \frac{-1\pm\sqrt{1+8}}{2} = \frac{-1\pm 3}{2} \begin{cases} 1 \\ -2 \end{cases}$

$$3x^2-6 = A(x-1)(x+2) + Bx(x+2) + Cx(x-1)$$

$x=0 \to -6=-2A \to A=3$

$x=1 \to -3=3B \to B=-1$

$x=-2 \to 6=6C \to C=1$

$(*)\ \displaystyle\int\left(x-1+\frac{3}{x}-\frac{1}{x-1}+\frac{1}{x+2}\right)dx = \frac{x^2}{2} + 3\,\mathrm{Ln}|x| - \mathrm{Ln}|x-1| + \mathrm{Ln}|x+2| + k$

10) $\displaystyle\int \mathrm{Ln}\,x\,dx = \begin{pmatrix} u=\mathrm{Ln}\,x & du=\frac{1}{x}dx \\ dv=dx & v=x \end{pmatrix} = x\,\mathrm{Ln}\,x - \int x\cdot\frac{1}{x}\,dx =$

$= x\,\mathrm{Ln}\,x - x + k.$